Exercise 6 Let \( L = (V, C, R) \) with \( V = \{w, y\} \), \( C = \{d, e\} \) and \( R = \{r, s\} \) where \( r \) has arity 1 and \( s \) has arity 2. Which of the following are atoms over \( L \)? Which are ground atoms? Justify your answers.

(a) \( d(w, w) \)  
(b) \( r(d, e) \)  
(c) \( s(w, w) \)  
(d) \( r(y) \)

Exercise 7 Let \( L = (V, C, R) \) with \( V = \{x, y\} \), \( C = \{\text{barack, michelle, craig, malia}\} \) and \( R = \{\text{motherOf, parentOf, grandmotherOf}\} \), all with arity 2.

Which of the Datalog facts (1) to (9) from Example 1.1.1 are atoms over \( L \)? Justify your answers.

Exercise 8 Write a Datalog program which captures the following natural language sentences.

(a) If somebody is an orphan, then all his parents are dead.
(b) Every orphan is a human being.
(c) Somebody’s father is also that person’s parent.
(d) Harry Potter is an orphan.
(e) James Potter is the father of Harry Potter.

Exercise 9 Give three distinct Herbrand interpretations for the following Datalog program, where \( a, b \) are constants.

\[
\begin{align*}
q(a) \\
p(b) \\
q(x) & \rightarrow p(x) \\
q(y) \land p(y) & \rightarrow r(b)
\end{align*}
\]

Exercise 10 Evaluate the following.

(a) \( (p(x, y, x) \land q(x, y, y) \land r(y, y) \rightarrow t(x))[x/a, y/b] = \ldots \)
(b) \( (p(x) \land q(x) \rightarrow r(x))[x/c][x/d] = \ldots \)
(c) \( (q(a, x) \land p(x, y) \land q(y, a) \rightarrow r(y))[x/a][x/b] = \ldots \)
(d) \( (p(x, x) \land q(x, y) \rightarrow p(x, y))[y/b][y/c][x/b] = \ldots \)

Exercise 11 Which of the substitutions in Exercise 10 are ground substitutions?

Exercise 12 Give the grounding of the Datalog program from Exercise 9.

Exercise 13 Give a Herbrand model for the Datalog program in Exercise 9.

Exercise 14 Give three distinct Herbrand models for the Datalog program \( P \) consisting of the following rules.

\[
\begin{align*}
p(a, b) \\
q(c) \\
p(x, y) & \rightarrow q(x)
\end{align*}
\]