CS 2210: Logic for Computer Scientists
Description Logics

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The Semantic Web Stack

User Interface & applications

Trust

Proof

Unifying Logic

Query: SPARQL

ontology: OWL

Rules: RIF

RDF-S

Data interchange: RDF

XML

URI

Unicode

Crypto
Web Ontology Language (OWL)

- W3C Recommendation since 2004
- OWL 2 since 2009

- based on description logics
- essentially, a decidable fragment of first-order predicate logic
Description Logics (DLs)

<table>
<thead>
<tr>
<th>classes/concepts</th>
<th>unary predicates</th>
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<tbody>
<tr>
<td>A, B, C</td>
<td>A(x), B(X), C(x)</td>
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<th>roles/properties</th>
<th>binary predicates</th>
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<td>R, S</td>
<td>R(x,y), S(x,y)</td>
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<th>individuals</th>
<th>constants</th>
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Some DL constructors

class conjunction

\[ C \sqcup D \rightarrow C(x) \land D(x) \]

eexistential restriction

\[ \exists R. C \rightarrow 9y (R(x,y) \land C(y)) \]

class inclusion/subsumption

\[ C \sqsubseteq D \rightarrow C(x) \rightarrow D(x) \]

\[ C' \sqsubseteq D \rightarrow C(x) \rightarrow D(x) \]

role chains

\[ R_1 \pm ... \pm R_n \land R \rightarrow R_1(x,x_1) \land ... \land R_n(x_n,x_{n+1}) \rightarrow R(x,x_{n+1}) \]
Some DL constructors

ThaiDish v 9contains.Nut
Nutallergic u 9eats.Nut v Unhappy
eats ± contains v eats

ThaiDish(x) → 9y (contains(x,y) AE Nut(y))
Nutallergic(x) AE 9y (eats(x,y) AE Nut(y)) → Unhappy(x)
eats(x,y) AE contains(y,z) → eats(x,z)
Some DL constructors

ThaiDish \lor 9 \text{contains} . \text{Nut}

\text{Nutallergic} \lor 9 \text{eats} . \text{Nut} \lor \text{Unhappy}

eats \pm \text{contains} \lor \text{eats}

inverse roles

\begin{align*}
R & \quad S^- \\
R(x,y) & \quad S(y,x)
\end{align*}

This logic is already undecidable! (see e.g. [ISWC 2007])

Name of the logic: ELRI
Decidability is a central characteristics of description logics.
Retaining Decidability

1. Disallow 9:
   Essentially leads to OWL RL.
   Fragment of Datalog.
   Tractable (i.e., polynomial complexity).

2. Disallow inverse roles:
   Essentially leads to OWL EL.
   Akin “in spirit” to existential rules/Datalog+-.
   Tractable.

3. Restrict recursion in role chains (a.k.a. regularity restriction):
   With further constructors, leads to OWL DL, a.k.a. SROIQ.
   Decidable, but not tractable.
Further essential DL constructors

The following can be used in OWL EL (logic remains tractable).

**Self**

\[ C \lor \exists R. \text{Self} \quad C(x) \rightarrow R(x,x) \]

Can be used e.g. for typecasting.

**Nominals**

\[ \{a\} \lor C \]

\[ C(a) \quad \text{a is a constant} \]

\[ C(v) \lor \{a\} \]

\[ C(x) \rightarrow x = a \]

\[ \{a\} \\backslash \{b\} \]

\[ \rightarrow a = b \]

\[ A \land \exists R.\{b\} \subseteq C \text{ becomes } A(x) \land R(x, b) \rightarrow C(x) \]
Further essential DL constructors

The following are used in expressive (intractable) DLs

class negation

\( C \) : \( C(x) \)

class disjunction

\( C \sqcup D \) : \( C(x) \sqcup D(x) \)

universal restriction

\( 8R.C \) : \( 8y \ (R(x,y) \rightarrow C(y)) \)

There are some more of course.
Rules in OWL

\[
A \subseteq B \text{ becomes } A(x) \rightarrow B(x)
\]

\[
R \subseteq S \text{ becomes } R(x, y) \rightarrow S(x, y)
\]

\[
A \sqcap \exists R. \exists S. B \subseteq C \text{ becomes } A(x) \land R(x, y) \land S(y, z) \land B(z) \rightarrow C(x)
\]

\[
R \circ S \subseteq T \text{ becomes } R(x, y) \land S(y, z) \rightarrow T(x, z)
\]

\[
A \sqcap \exists R. \{b\} \subseteq C \text{ becomes } A(x) \land R(x, b) \rightarrow C(x)
\]
Rolification

\[ \text{Elephant}(x) \land \text{Mouse}(y) \rightarrow \text{biggerThan}(x, y) \]

- Rolification of a concept A: \[ A \in R_A \cdot \text{Self} \]

\[ \text{Elephant} \equiv \exists R_{\text{Elephant}} \cdot \text{Self} \]
\[ \text{Mouse} \equiv \exists R_{\text{Mouse}} \cdot \text{Self} \]
\[ R_{\text{Elephant}} \circ U \circ R_{\text{Mouse}} \sqsubseteq \text{biggerThan} \]
worksAt(x, y) \land University(y) \land supervises(x, z) \land PhDStudent(z) 
\rightarrow professorOf(x, z)

R_{\exists worksAt. University \circ supervises \circ R_{PhDStudent}} \subseteq professorOf.
Tree-shaped rules

\[ R_1(x, y) \land C_1(y) \land R_2(y, w) \land R_3(y, z) \land C_2(z) \land R_4(x, x) \rightarrow C_3(x) \]

\[ \exists R_1. (C_1 \cap \exists R_2. C_1 \cap \exists R_3. C_2) \cap \exists R_4. Self \subseteq C_3 \]
Acyclic Rules

\[ R_1(y, x) \land C_1(y) \land R_2(w, y) \land R_3(y, z) \land C_2(z) \land R_4(x, x) \rightarrow C_3(x) \]

\[ \exists R_1^- . (C_1 \sqcap \exists R_2^- . \top \sqcap \exists R_3 . C_2) \sqcap \exists R_4 . Self \sqsubseteq C_3 \]
So how can we pinpoint this?

- Tree-shaped bodies
- First argument of the conclusion is the root

\[ C(x) \land R(x,a) \land S(x,y) \land D(y) \land T(y,a) \land E(x) \]
- \( C \cup 9R.\{a\} \cup 9S.\{D \cup 9T.\{a\}\} \land E \)

Duplicating nominals is ok
Rule bodies as graphs

\[ C(x) \land R(x, a) \land S(x, y) \land D(y) \land T(y, a) \rightarrow P(x, y) \]

\[ a_1 \leftarrow x \rightarrow y \rightarrow a_2 \]

\[ C \cup 9R.\{a\} \lor 9R1.\text{Self} \]
\[ D \cup 9T.\{a\} \lor 9R2.\text{Self} \]
\[ R1 \pm S \pm R2 \lor P \]
So how can we pinpoint this?

- Tree-shaped bodies
- First argument of the conclusion is the root

\[ C(x) \land R(x,a) \land S(x,y) \land D(y) \land T(y,a) \land V(x,y) \]

\[
\begin{align*}
C \cup 9R.\{a\} & \lor 9R_1.\text{Self} \\
D \cup 9T.\{a\} & \lor 9R_2.\text{Self} \\
R_1 \circ S \circ R_2 & \lor V
\end{align*}
\]
Rule bodies as graphs

\[\text{hasReviewAssignment}(v, x) \land \text{hasAuthor}(x, y) \land \text{atVenue}(x, z) \]
\[\land \text{hasSubmittedPaper}(v, u) \land \text{hasAuthor}(u, y) \land \text{atVenue}(u, z)\]
\[\rightarrow \text{hasConflictingAssignedPaper}(v, x)\]

with \(y, z\) constants (otherwise not expressible):

\[R_{\exists\text{hasSubmittedPaper}. (\exists\text{hasAuthor}. \{y\} \land \exists\text{atVenue}. \{z\})} \circ \text{hasReviewAssignment} \]
\[\circ R_{\exists\text{hasAuthor}. \{y\} \land \exists\text{atVenue}. \{z\}} \]
\[\sqsubseteq \text{hasConflictingAssignedPaper}\]
Conclusions

Description Logics

• Adopted as Semantic Web standard OWL
• Essentially, a fragment of first-order predicate logic, but decidable.

• Acyclic rules can be encoded.
• Existential quantifiers can be used, but in a restricted fashion.

• Conditions play together such that decidability is retained.
References

A tutorial:


Background reading:


References